HEAT TRANSFER BETWEEN A SURFACE AND A FLUIDIZED BED

V. P. Myasnikov and M. S. Rozhdestvenskaya

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 6, pp. 139–145, 1968

One of the important applications of the fluidized bed is as a cooling agent in heat exchangers. Two principal theoretical explanations are offered for the heat transfer mechanism. The first is based on the idea of multiple transport of heat from the surface by the solid particles [1–3]; the second on the idea of "bunches" of particles that replace each other near the wall and are heated in the same way as a porous medium [4–9].

In paper we study heat transfer between a fluidized bed and a surface based on the kinetic model previously developed in [10–12].

1. **Formulation of the problem.** We assume that at time t a particle passes out of direct thermal contact with a surface at a fixed temperature T_w. Moreover, we denote by β_0 the coefficient of heat transfer between the particle and the wall. The amount of heat given up to the particle by the wall in time dt is

$$dQ_i = mc_i dT_i = \beta_0 (T_w - T_i)\, dt. \tag{1.1}$$

Here, m is the mass of the particle, c_i is its specific heat, and T_i is its volume-averaged temperature.

If the particle appeared in the zone of direct thermal contact with the cooled surface at time $\tau < t$, its temperature at time t is

$$T_i(t) = T_w - [T_w - T_i(\tau)] \exp\left[-\frac{\beta_0}{mc_i}(t-\tau)\right]. \tag{1.2}$$

We introduce a particle number distribution function $\psi(t, x, u, T_i)$ such that the average number of particles in a volume (x, x + dx) with velocities and temperatures in the intervals (u, u + du) and (T_i, $T_i + dT_i$) is equal to $\Psi\, dx\, du\, dT_i$.

The function ψ is related with the Boltzmann distribution function in the usual way:

$$f(t, \mathbf{x}, \mathbf{u}) = \int_0^\infty \Psi(t, \mathbf{x}, \mathbf{u}, T_i)\, dT_i.$$

For particles that entered the thermal contact zone at time τ, the average temperature rise is

$$T_i{}^\circ = T_w - [T_w - \langle T_i(\tau)\rangle] \exp\left[-\frac{\beta_0}{mc_i}(t-\tau)\right],$$

$$\langle T_i(\tau)\rangle = \frac{1}{n}\iint_0^\infty T_i \Psi(\tau, \mathbf{x}, \mathbf{u}, T_i)\, dT_i\, d\mathbf{u}. \tag{1.3}$$

We denote by $P(\xi)$ the probability that a particle will remain in the thermal contact zone for a period of time ξ. Then the most probable temperature of the particles that leave the thermal contact zone at time t is

$$\langle T_i{}^\circ\rangle = T_w - \int_{-\infty}^{t} [T_w - \langle T_i(\tau)\rangle]\, P(t-\tau) \exp\left[-\frac{\beta_0}{mc_i}(t-\tau)\right] d\tau. \tag{1.4}$$

The amount of heat removed by these particles from the thermal contact zone at time t is equal to

$$Q_n{}^* = mc_i \left\{T_w - \int_{-\infty}^{t} [T_w - \langle T_i(\tau)\rangle]\, P(t-\tau) \exp\left[-\frac{\beta_0}{mc_i}(t-\tau)\right] d\tau\right\} \int_{(\mathbf{un})>0} (\mathbf{un})\, f\, d\mathbf{u}, \tag{1.5}$$

where **n** is the normal to the cooled surface directed into the bed.

The amount of heat carried by the particles from the interior of the bed into the thermal contact zone at the same moment of time t

$$Q_n^{\circ} = -mc_i \langle T_i(t)\rangle \int\limits_{(\mathbf{un})<0} (\mathbf{un}) f \, d\mathbf{u}\,. \tag{1.6}$$

Now considering that at an impermeable surface

$$\int\limits_{(\mathbf{un})>0} (\mathbf{un}) f \, d\mathbf{u} + \int\limits_{(\mathbf{un})<0} (\mathbf{un}) f \, d\mathbf{u} = 0 \tag{1.7}$$

for the quantity of heat removed from the thermal contact zone at time t we find

$$q_n = mc_i \left\{T_w - \langle T_i(t)\rangle - \int\limits_{-\infty}^{t} [T_w - \langle T_i(\tau)\rangle] P(t-\tau) \exp\left[-\frac{\beta_0}{mc_i}(t-\tau)\right] d\tau\right\} \times \\ \times \int\limits_{(\mathbf{un})>0} (\mathbf{un}) f \, d\mathbf{u}. \tag{1.8}$$

The quantity q_n must be equal to the flow of heat transported by the particles in the fluidized bed, so that in accordance with the results of [11] we have

$$-\frac{3mc_i}{8\sigma^2\chi}\left(\frac{\theta}{m\pi}\right)^{1/2}\frac{\partial\langle T_i\rangle}{\partial n} = mc_i\left\{T_w - \langle T_i(t)\rangle - \int\limits_{-\infty}^{t} [T_w - \langle T_i(\tau)\rangle] P(t-\tau) \exp \times \right. \\ \left. \times \left[-\frac{\beta_0}{mc_i}(t-\tau)\right] d\tau\right\} \int\limits_{(\mathbf{un})>0} (\mathbf{un}) f \, d\mathbf{u}\,, \\ \chi = \frac{1 - {}^{11}/_{16} N}{1-N}, \qquad v_* n = N. \tag{1.9}$$

Here, σ is the particle diameter, θ is the pseudo temperature [10], v_* is the volume per particle, assuming close packing, and n is the average number of particles per unit volume.

As may be seen from (1.9) the instantaneous heat flow from the cooled surface at time t depends on the entire prehistory of the process. Only in the steady state, when $\langle T_i\rangle$ does not depend on time, do we have

$$-\frac{3mc_i}{8\sigma^2\chi}\left(\frac{\theta}{m\pi}\right)^{1/2}\frac{\partial\langle T_i\rangle}{\partial n} = h(T_w - \langle T_i\rangle)\,, \\ h = mc_i\left\{1 - \int\limits_0^{\infty} P(\xi)\exp\left(-\frac{\beta_0\xi}{mc_i}\right)d\xi\right\}\int\limits_{(\mathbf{un})>0} (\mathbf{un}) f \, d\mathbf{u}\,. \tag{1.10}$$

2. **Steady-state cooling of a vertical surface.** We will now examine the characteristics of the steady-state process of heat transfer between a vertical surface and a fluidized bed.

For simplicity, we assume that the longitudinal extent of the bed is large compared with its thickness, that there are no macroscopic particle flows in the bed, and that the average velocity of the pseudogas is equal to zero.

The equations of heat transfer in the bed [12] have the following form:

$$\lambda_i \Delta\langle T_i\rangle + \lambda_i\left(\frac{\partial\varphi}{\partial q}\right)_0\left(\frac{\partial\langle T_f\rangle}{\partial y} - \frac{\partial\langle T_i\rangle}{\partial y}\right) + mnc_i\varphi(\langle T_f\rangle - \langle T_i\rangle) = 0\,, \\ \lambda_f\frac{\partial^2\langle T_f\rangle}{\partial y^2} - \rho_f c_f Q\frac{\partial\langle T_f\rangle}{\partial y} - mnc_i\varphi(\langle T_f\rangle - \langle T_i\rangle) = 0. \tag{2.1}$$

The coordinate system has been selected so that the x-axis is directed normal to the cooled wall and lies in the plane of the distributor supporting the bed, while the y-axis is directed vertically upwards.

The effective thermal conductivities λ_i and λ_f were obtained in [11]; the dependence of the function φ on the flow velocity through the bed, the bed voidage and the thermal conductivities of the particles and the gas is discussed in [12].

Since the thermal regime of the bed is assumed steady, all the heat removed from the wall by the particles is removed from the bed by the gas flow. From the second equation of (2.1) in the general case

$$\frac{\rho_f Q c_f}{L} \sim \rho_i c_i \varphi, \tag{2.2}$$

where ρ_i is the density of the particle material and L the characteristic scale of flow-particle temperature equalization.

Taking into account the dependence of φ on the above-mentioned flow parameters [12] and denoting by l the thickness of the bed, we have

$$\frac{L}{l} \sim \frac{\nu_f}{\varkappa_f} \frac{\sigma^2 R}{l H_T(n, R, k_f/k_i)}, \qquad R = \frac{Q\sigma}{\nu_f}, \tag{2.3}$$

where ν_f is the kinematic viscosity of the gas, $\varkappa_f$ is its thermal diffusivity, k_f and k_i are the thermal conductivities of the gas and the particles, σ is the particle diameter, and H_T is a quantity proportional to the Nusselt numbers for particle-flow heat transfer.

In the general case $L \sim l$, but for slowly heated particles $L \gg l$.

Turning now to the first equation of (2.1), from the heat balance condition we find

$$\frac{\lambda_i}{\delta} L \sim \rho_f Q c_f \delta, \tag{2.4}$$

where δ is the characteristic thickness of the layer of heated particles in the vicinity of the cooled wall.

From (2.2) and (2.4) we obtain

$$\frac{\delta^2}{L^2} \sim \frac{Q}{\varphi L} \frac{\sigma}{L} \ll 1. \tag{2.5}$$

Since $L(\partial\varphi/\partial q)_0 \sim 1$, retaining in (2.1) the terms with the greatest order of magnitude, we have

$$\begin{gathered} \lambda_i \frac{\partial^2 \langle T_i \rangle}{\partial x^2} + mnc_i\varphi(\langle T_f \rangle - \langle T_i \rangle) = 0, \\ \lambda_f \frac{\partial^2 \langle T_f \rangle}{\partial y^2} - \rho_f Q c_f \frac{\partial \langle T_f \rangle}{\partial y} - mnc_i\varphi(\langle T_f \rangle - \langle T_i \rangle) = 0. \end{gathered} \tag{2.6}$$

The boundary conditions for (2.6) are conditions (1.10) and

$$\langle T_f \rangle|_{y=0} = T_s. \tag{2.7}$$

System (2.6) can easily be solved if $L \sim l$. We then represent $\langle T_i \rangle$ and $\langle T_f \rangle$ in the expanded form:

$$\langle T_i \rangle = \theta_i^{(0)} + \theta_i^{(1)} + \ldots, \qquad \langle T_f \rangle = T_s + \theta_f^{(1)} + \theta_f^{(2)} + \ldots. \tag{2.8}$$

Substituting (2.8) into (2.6), for the successive approximations we obtain

$$\begin{gathered} \lambda_i \frac{\partial^2 \theta_i^{(0)}}{\partial x^2} + mnc_i\varphi(\theta_f^{(0)} - \theta_i^{(0)}) = 0, \qquad \theta_f^{(0)} = T_s, \ \lambda_i \frac{\partial^2 \theta_i^{(1)}}{\partial x^2} + mnc_i\varphi(\theta_f^{(1)} - \theta_i^{(1)}) = 0, \\ \lambda_f \frac{\partial^2 \theta_f^{(1)}}{\partial y^2} - \rho_f Q c_f \frac{\partial \theta_f^{(1)}}{\partial y} - mnc_i\varphi(\theta_f^{(1)} + \theta_f^{(0)} - \theta_i^{(0)}) = 0. \end{gathered} \tag{2.9}$$

The solution of the first equation of (2.9) satisfying boundary condition (1.10) at x = 0 is as follows:

$$\theta_i^{(0)} = T_s + \frac{h}{h + \lambda_i k}(T_w - T_s)e^{-kx}, \qquad k = \left(\frac{mnc_i\varphi}{\lambda_i}\right)^{1/2}. \tag{2.10}$$

The equation for the first approximation of the temperature field of the suspending flow is then written in the form:

$$mnc_i\varphi\theta_f^{(1)} + \lambda_f \frac{\partial^2 \theta_f^{(1)}}{\partial y^2} - \rho_f Q c_f \frac{\partial \theta_f^{(1)}}{\partial y} = mnc_i\varphi \frac{h}{h + \lambda_i k}(T_w - T_s)e^{-kx}, \tag{2.11}$$

and the corresponding solution of (2.11) with account for condition (2.7) is

$$\theta_f^{(1)} = \frac{h}{h+\lambda_i k}(T_w - T_s)\,e^{-kx}(1-e^{-sy}),$$

$$s = \frac{1}{2\lambda_f}\{[(\rho_f Q c_f)^2 + 4\lambda_f mnc_i\varphi]^{1/2} - \rho_f Q c_f\}. \tag{2.12}$$

The equation for the first approximation of the temperature field of the pseudo gas is as follows:

$$\lambda_i \frac{\partial^2\theta_i^{(1)}}{\partial x^2} - mnc_i\varphi\theta_i^{(1)} = -mnc_i\varphi\frac{h}{h+\lambda_i k}(T_w - T_s)\,e^{-kx}(1-e^{-sy}). \tag{2.13}$$

By solving it we can obtain the distribution of the temperature field of the pseudogas. After simple transformations we obtain

$$\langle T_f\rangle = T_s + \frac{h}{h+\lambda_i k}(T_w - T_s)(1-e^{-sy})\,e^{-kx} + \ldots,$$

$$\langle T_i\rangle = T_s + \frac{h}{h+\lambda_i k}(T_w - T_s)\,e^{-kx} + \frac{{}^1/_2\,kh}{h+\lambda_i k}(T_w - T_s)(1-e^{-sy})\,xe^{-kx} + \tag{2.14}$$

$$+ \frac{{}^1/_2\,\lambda_i kh}{(h+\lambda_i k)^2}(T_w - T_s)(1-e^{-sy})\,e^{-kx} + \cdots.$$

We note that if $h > \lambda_i k = (mnc_i\varphi\lambda_i)^{1/2}$, the maximum rate of heat transfer between components is reached at

$$x = \frac{1}{k}\frac{h-\lambda_i k}{h+\lambda_i k}. \tag{2.15}$$

3. **Heat transfer coefficient.** By definition the local coefficient of heat transfer between the bed and the surface is

$$\alpha = h\left[\frac{T_w - \langle T_i\rangle}{T_w - T_s}\right]_{x=0} = h\frac{k\lambda_i}{h+k\lambda_i}\left[\frac{k\lambda_i + {}^1/_2\,h}{k\lambda_i + h} + \frac{{}^1/_2\,h}{k\lambda_i + h}e^{-sy}\right]. \tag{3.1}$$

We note that its value falls with increase in the distance from the distributor. This result is confirmed by experiment.

Within the limits of accuracy of the approximation

$$\alpha_{max} = h\frac{k\lambda_i}{h+k\lambda_i}, \qquad \alpha_{min} = \alpha_{max}\frac{k\lambda_i + {}^1/_2\,h}{k\lambda_i + h}. \tag{3.2}$$

Denoting by d the length of the surface cooled by the bed, for the average value of the heat transfer coefficient α_0 we obtain

$$\alpha_0 = \frac{1}{d}\int_0^d \alpha(y)\,dy = \alpha_{max}\left[\frac{k\lambda_i + {}^1/_2\,h}{k\lambda_i + h} + \frac{{}^1/_2\,h}{k\lambda_i + h}\cdot\frac{(1-e^{-sd})}{sd}\right]. \tag{3.3}$$

It follows from (2.12) and (2.3) that $s \sim 1/L$. For sufficiently slowly heated particles $d \ll L$, so that in this case

$$\alpha_0 \approx \alpha_{max}. \tag{3.4}$$

As has been pointed out (see [13, 14]), for aluminum particles $L \sim 2-4$ cm. For sand particles, in view of their much lower thermal conductivity, the values of L will be several times greater.

From (3.4) and (1.10) we obtain an expression for α_0

$$\alpha_0 = mnc_i w_n A\frac{k\lambda_i}{k\lambda_i + mnc_i w_n A},$$

$$A = 1 - \int_0^\infty P(\xi)\exp\left[-\frac{\beta_0\xi}{mc_i}\right]d\xi, \qquad w_n = \left(\frac{\theta}{2m\pi}\right)^{1/2}, \tag{3.5}$$

where θ is the pseudo temperature [10].

Using the expression for k from (2.10) and for λ_i from [11], we can show that with an accuracy sufficient for practical purposes

$$\frac{k\lambda_i}{k\lambda_i + mnc_i w_n A} \approx 1. \tag{3.6}$$

Therefore, finally,

$$\alpha_0 = mnc_i \left(\frac{\theta}{2\pi m}\right)^{1/2} \left\{1 - \int_0^\infty P(\xi) \exp\left[-\frac{\beta_0 \xi}{mc_i}\right] d\xi\right\}. \tag{3.7}$$

In first approximation the parameters β_0 and $P(\xi)$ in (3.7) can be determined as follows. Assuming that the heat transfer between a particle and the wall is realized through an intermediate layer of gas of a certain average thickness δ_0, we obtain

$$\beta_0 = \frac{k_f}{\delta_0} \frac{\pi\sigma^2}{4}. \tag{3.8}$$

We also assume that the time spent by a particle in the zone of direct thermal contact with the wall coincides with twice its mean free time (the average interval between the collision that propels the particle towards the wall and the next collision after it strikes the wall). Then

$$\lambda = \sigma\left(\frac{1}{N}\right)^{1/3}, \qquad N = nv_*, \qquad P(\xi) = \delta\left(\xi - \frac{2\lambda}{w_n}\right). \tag{3.9}$$

Substituting (3.8) and (3.9) into (3.7), we finally obtain

$$\alpha_0 = mnc_i \left(\frac{\theta}{2\pi m}\right)^{1/2} \left\{1 - \exp\left[-\frac{\pi k_f \sigma^3}{2\delta_0 mc_i N^{1/3}} \left(\frac{2m\pi}{\theta}\right)^{1/2}\right]\right\}. \tag{3.10}$$

An expression for θ was obtained in [10]:

$$\theta = \frac{mD}{3}\omega^2 \left\{\frac{1}{1-\omega N} + \frac{1}{\omega}\frac{\partial \ln \Phi}{\partial N}\right\}^2 \frac{N^2 (1-N)^2 Q^2}{(1-\omega N)^2 (1 - {}^{17}/_{32} N^2 + {}^{11}/_{16} N^3 - N)}, \tag{3.11}$$
$$\omega = \pi\sigma^3 / 6v_*.$$

Here, Φ is the Stokes drag coefficient of the particle calculated for unit mass, and Q is the velocity of the suspending gas flow in the free cross section of the apparatus.

In the steady state a certain relationship exists between Q and N [15]. If we use in the calculations the relation for Φ proposed in [16], then

$$Q = \frac{\nu_f}{\sigma} \frac{A_r (1-\omega N)^{4.75}}{18 + 0.6 [A_r (1-\omega N)^{4.75}]^{1/2}}, \qquad A_r = \frac{g\sigma^3}{\nu_f^2} \frac{\rho_i}{\rho_f}. \tag{3.12}$$

The graphs in the figure give the results of calculating α_0, as a function of Q from formulas (3.10)–(3.12) for experiments with quartz sand [17], in which the conditions of validity of the relations obtained were satisfied with sufficient accuracy. In carrying out the calculations it was assumed that $\omega = 0.6$, $D = 10^{-2}$, $\delta_0 = 0.8\delta$. The curves correspond to the theoretical relations, while the experimental points correspond to the data of [17, 18]. An analysis of the results obtained indicates that the theoretical results not only lead to good qualitative agreement with experiment but also give satisfactory quantitative agreement.

At large gas velocities the behavior of the graph of α_0 (kcal/m^2 · hr · deg) versus Q (m/sec) is also in good qualitative agreement (see figure) with the known experimental data [18]. In this case the experiments were performed on glass spheres, whose thermophysical properties differ from the corresponding properties of quartz sand, although the difference is not very considerable. The experimental points corresponding to this case for particles whose diameters are close to those used in [17] have been plotted in Fig. 1 in accordance with the data of [18]. The lower position of the experimental points from [18] in Fig. 1 for particles with similar diameters is associated with the much greater nonuniformity of the particle distribution over the height of the bed as compared with [17]. The maximum of α_0 increases with decrease in particle diameter; the curves in the figure correspond to particles with diameters of

$3.15 \cdot 10^{-2}$, $4.5 \cdot 10^{-2}$, and $7.5 \cdot 10^{-2}$ cm for quartz sand 1, 2, and 3 and $8.5 \cdot 10^{-2}$ and $4.5 \cdot 10^{-2}$ cm for glass spheres 4 and 5.

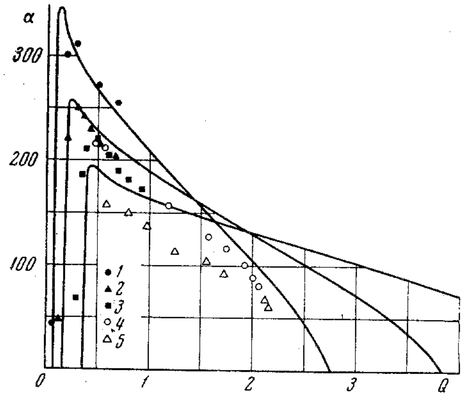

Fig. 1

The authors thank V. G. Levich for his interest and valuable comments.

REFERENCES

1. C. van Heerden, A. P. P. Nobel, and D. W. Krevelen, "Mechanism of heat transfer in fluidized beds," Industr. Engng. Chem., vol. 45, no. 6, pp. 1237–1242, 1953.

2. H. S. Mickley and D. F. Fairbanks, "Mechanism of heat transfer to fluidized bed," A. J. Ch. E. Journal, vol. 1, no. 3, pp. 374–384, 1955.

3. S. S. Zabrodskii, "Analysis of experimental data on heat transfer in fluidized beds," IFZh, vol. 1, no. 4, pp. 22–30, 1958.

4. R. Ernst, "Der Mechanismus des Wärmeübergangs an Wärmeaustauscher in Fliessbetten (Wirbelschichten)," Chem. Ing. Techn., vol. 31, no. 3, pp. 166–173, 1959.

5. E. Rukenshtein, "The mechanism of heat and mass transfer in fluidized beds," Zh. prikl. khimii, vol. 35, no. 1, pp. 70–80, 1962.

6. H. Reuter, "Mechanismus der Blasen im Gas–Feststoff–Fließbett," Chem Ing. Techn., vol. 35, no. 3, pp. 219–228, 1963.

7. O. M. Todes and A. K. Bondareva, "Heat transfer from a fluidized bed to a heat transfer surface," collection: Applications of the Fluidized Bed in the National Economy [in Russian], Central Sci. Res. Inst. Inform. and Tech.-Econ. Research in Nonferrous Metallurgy, Moscow, 1965.

8. A. P. Baskakov, "Mechanism of heat transfer between a fluidized bed and a surface," vol. 6, no. 11, pp. 20–25, 1963.

9. N. I. Gal'perin, V. G. Ainshtein, and A. V. Zaikovskii, "Mechanism of heat transfer between a surface and an inhomogeneous granular fluidized bed," Khim. prom-st, no. 6, 1966.

10. V. P. Myasnikov, "Dynamic equations of motion of two-component systems," PMTF [Journal of Applied Mechanics and Technical Physics], no. 2, 1967.

11. V. P. Myasnikov, "Heat and mass transfer processes in fluidized beds," Proc. 3-rd All-Union Conference on Heat and Mass Transfer, vol. 9, [in Russian], Minsk, 1968.

12. V. P. Myasnikov, "Kinetic model of heat transfer processes in a fluidized bed," Izvestiya AN SSSR. ZHhG [Fluid Dynamics], no. 4, 1967.

13. N. I. Syromyatnikov, L. K. Vasanova, and Yu. N. Shimanskii, Heat and Mass Transfer in the Fluidized Bed [in Russian], Khimiya, Moscow, 1967.

14. L. K. Vasapova, "Thickness of the active zone in a fluidized bed," Tsvetn. metally, no. 2, 1965.

15. V. P. Myasnikov, "Distribution of suspended particles in a fluidized bed," PMTF [Journal of Applied Mechanics and Technical Physics], no. 3, 1968.

16. V. D. Goroshko, B. R. Rozenbaum, and O. M. Todes, "Approximate laws of hydraulics of a suspended bed and constrained fall," Izvestiya VUZ, Neft i gaz, no. 1, 1958.

17. E. Wicke and F. Fetting, "Wärmeübertragung in Gaswirbelschichten," Chem.–Ing. Techn., vol. 26, no. 6, pp. 301–309, 1954.

18. H. S. Mickley and Ch. A. Trilling, "Heat transfer characteristics of fluidized beds," Industr. Engng. Chem., vol. 41, pp. 1135–1147, 1949.

7 May 1968

Moscow